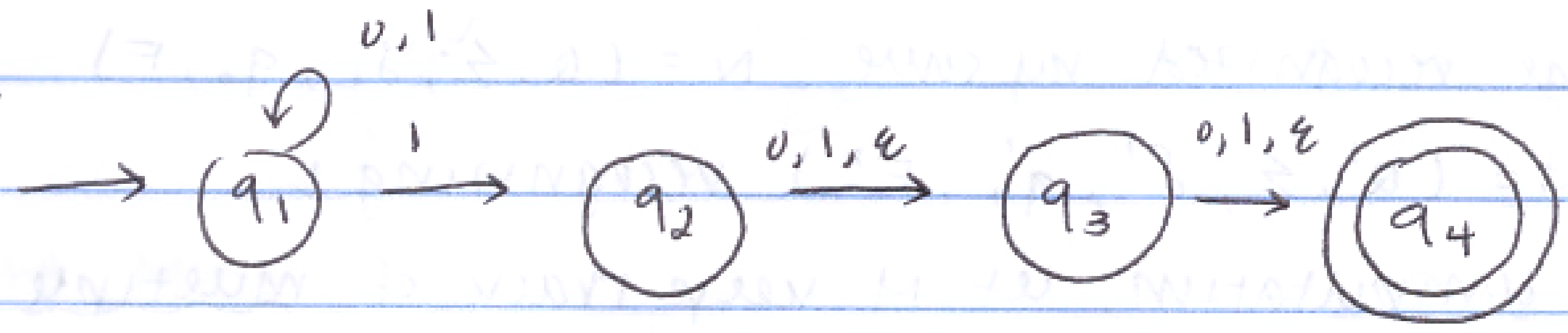
Last Class

$L(N_2') = \{x \mid x \text{ has } 1 \text{ in one of its last } 3 \text{ positions}\}$

NFA N_2'



NFA (Formal Definition): An NFA is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where:

1) Q is a finite set of states
2) Σ is a finite set called the alphabet.
3) $\delta : Q \times \Sigma_\varepsilon \longmapsto P(Q) \leftarrow$ power set of Q
   $\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$
4) q_0 is start state.
5) $F \subseteq Q$ are the accept states

e.g. For N_2'

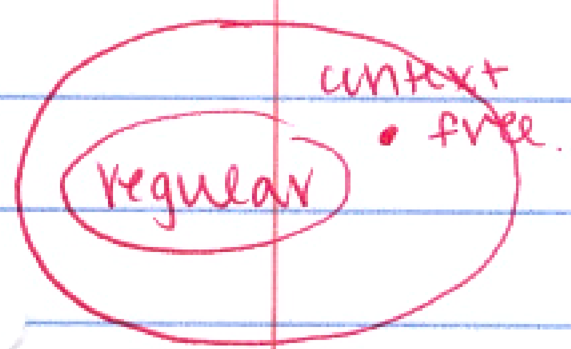
δ	0	1	ε
q_1	$\{q_1\}$	$\{q_1, q_2\}$	$\emptyset$
q_2	$\{q_3\}$	$\{q_3\}$	$\{q_3\}$
q_3	$\{q_4\}$	$\{q_4\}$	$\{q_4\}$
q_4	$\emptyset$	$\emptyset$	$\emptyset$

Formal definition of an acceptance for NFA

Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA and $w \in \Sigma^*$ a string: Then N accepts w if we can write $w = y_1 y_2 \dots y_n$ for $y_i \in \boxed{\Sigma_\varepsilon}$ $\forall\, 1 \le i \le m$, and $\exists$ sequence of states $(r_0, r_1, r_2, \dots, r_m) \in Q^{m+1}$ s.t.

1) $r_0 = q_0$ (start state is q_0)
2) $r_{i+1} \boxed{\in} \delta(r_i, y_{i+1})$ for $i \in \{0, \dots, m\}$ (follow transition function)
3) $r_m \in F$ (end in accept state)

"Q: DFA = NFA?" $\Leftrightarrow$ True that $\forall$ languages L,
$\exists$ DFA recognizing L iff
$\exists$ NFA recognizing L iff?



Trivial: If $\exists$ DFA recognizing L, then $\exists$ NFA recognizing L. (just don't use parallelism)

1.39

Theorem For every NFA can simulate by a DFA.

PF/ Let L be a language recognized by NFA $N = (Q, \Sigma, \delta, q_0, F)$

We construct DFA $M = (Q', \Sigma, \delta', q_0', F')$ recognizing L.

idea: At each point in computation, let M keep track of multiple states

$\therefore$ $Q' = P(Q)$, $|Q'| = 2^{|Q|}$

↳ exponential blowup!

FORMALLY Assume [for now] that N has no ε transitions

Set: (1) $Q' = P(Q)$

(3) $q_0' = \{q_0\}$ (4) $F' = \{R \in Q' \mid R$ contains an accept state of $N\}$

→ $\{q_0\}$

(2) For $R \in Q'$ and $a \in \Sigma$, set $\delta'(R, a) = \{q \in Q \mid q \in \delta(r, a)$ for some $r \in R\}$

R

$\{q_1, q_5\} \xrightarrow{a} \{\ ?\ \}$

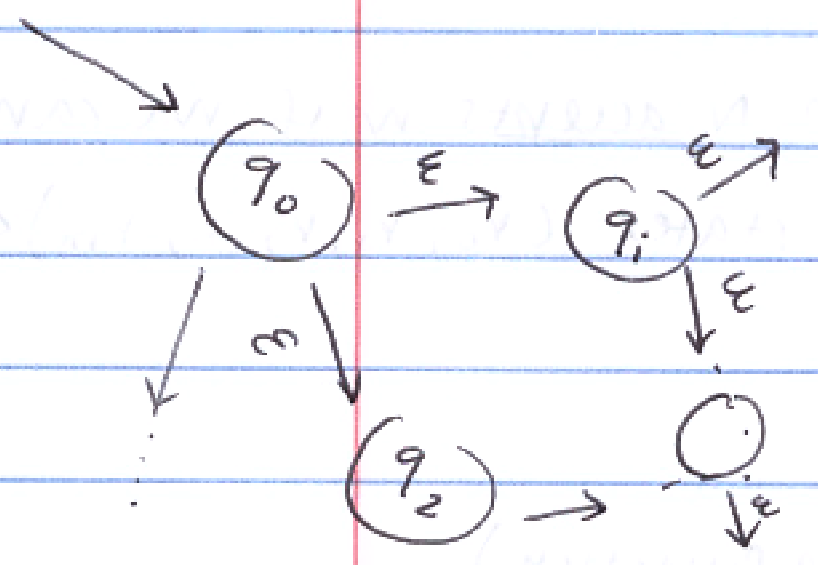
$\subset$ all possible states

Now let's add in ε transitions!

update: $\delta'(R, a) = \{q \in Q \mid q \in E(\delta(r, a))$ for some $r \in R\}$

↳ $E(S) = \{q \in Q \mid q$ can be reached from S by following ε transitions$\}$

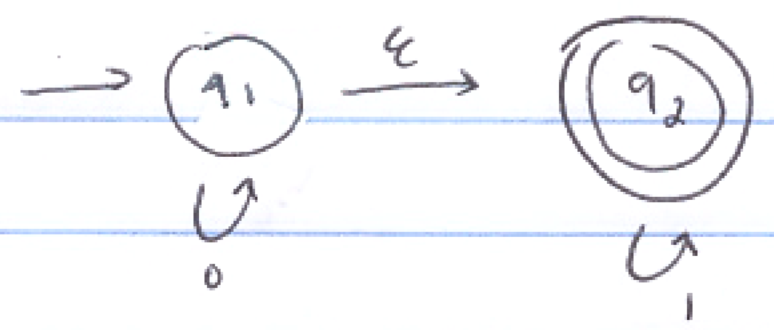
[$\exists$ ε-transitions]



Q: What if there exists ε-transitions from q_0?

update:

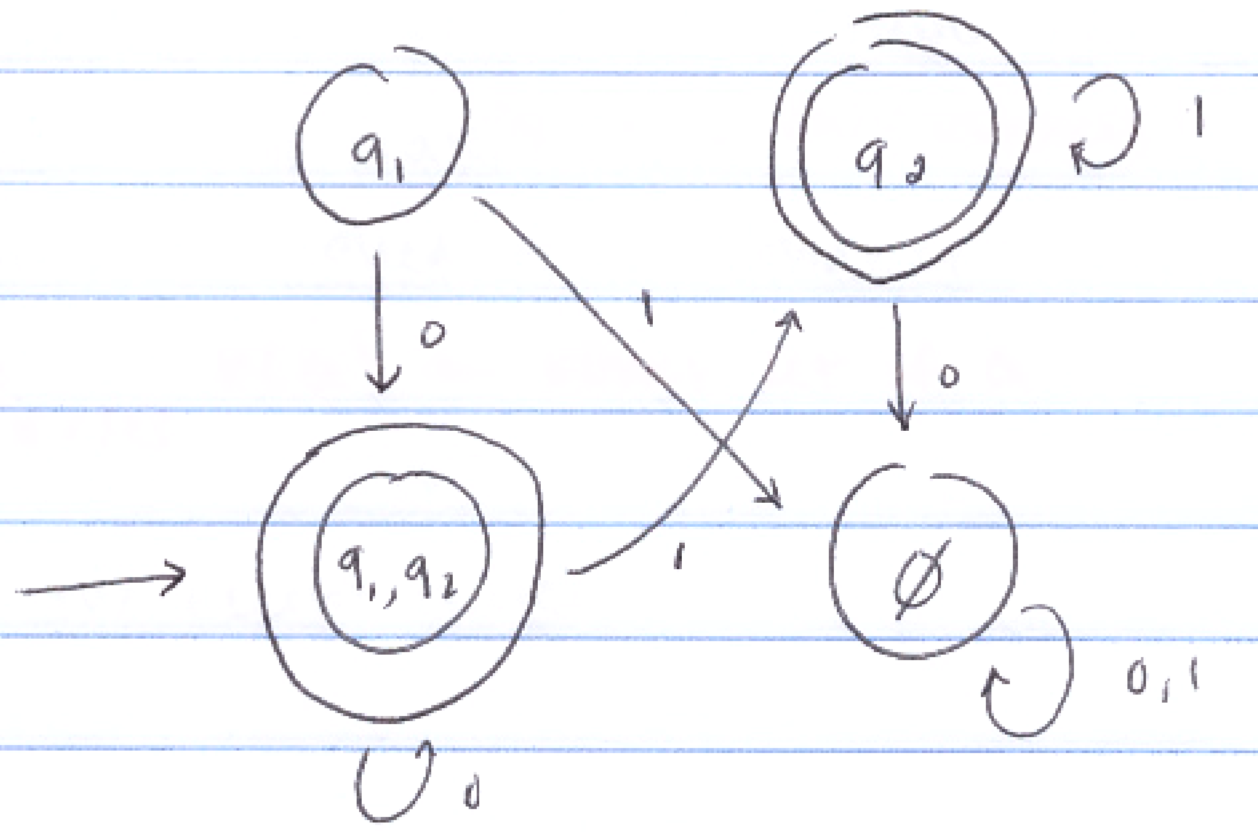
set $q_0' = \{E(q_0)\}$ ■

e.g. NFA N 

$L(N) = \{x \mid x$ doesn't contain a 1 before a 0$\}$

$x = 0^m 1^n,\ m,n \geq 0$

DFA M

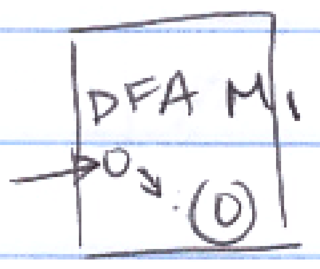


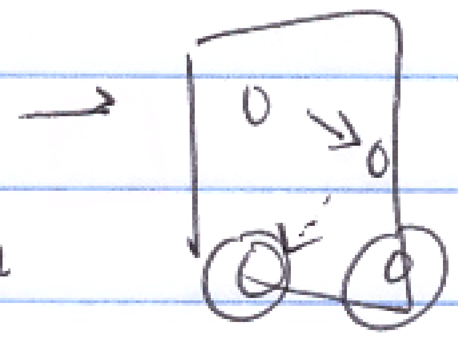
$\{q_1, q_2\} \xrightarrow{0} \{q_1, q_2\} \cup \emptyset$

$= \{q_1, q_2\}$

conclusion: DFAs & NFAs are equivalent in power!

e.g. closure under the union: $A \cup B = \{x \mid x \in A$ or $x \in B\}$

A $\rightarrow$ 

B DFA M_2 $\rightarrow$ 

$\Rightarrow$ design NFA N to recognize $A \cup B$ $\rightarrow$